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A Semi-Analytical Model to Calculate Energy Production in Single Fracture Geothermal Reservoirs

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ABSTRACT

The ultimate goal of geothermal reservoir management is to mine as much energy as possible from the reservoir. Since most of geothermal reservoirs are fractured, the development of improved techniques for monitoring, prediction, and control of two-phase mass and energy transport in the fractures are needed. Tracer testing is a powerful tool for characterizing fractures properties such as swept pore volume. By incorporating the pore volume of the fracture estimated from tracer data with a semi-analytical model of mass and energy transport in the fracture and matrix, the energy production rate can be calculated. Previous models to predict energy production assumed single-phase flow and semi-infinite matrix blocks and thus overestimate the energy that can be usefully produced from the reservoir. In this paper, a semi-analytical solution to the mass and energy balance equations is derived for two-phase flow in a fracture with heat conduction from a finite matrix. Calculations using the semi-analytical model are shown to be in good agreement with numerical simulations using the TETRAD simulator.

Introduction

Cold liquid water injection is a common practice in geothermal reservoirs for a variety of reasons such as pressure maintenance and spent water disposal. One of the goals of this research is to develop an improved method to predict the energy production from geothermal reservoirs where cold liquid injection is used. A suitable numerical simulator such as TETRAD is a useful tool for this purpose. In this paper, we develop and illustrate a semi-analytical model for the energy production from a fractured reservoir and compare it with numerical simulation results. This semi-analytical model, when used in conjunction with fracture characteristics information inferred from tracer return profiles, can be used to predict energy production from the reservoir. This is not thought to replace detailed simulation studies, since the true level of heterogeneity cannot be captured in the semi-analytical solution, but serves an important role in scoping studies or support of other engineering analyses.

Several authors (Lauwerier, 1955; Pruess and Bodvarsson, 1984; Kocabas, 2004) have derived analytical solutions for the problem of coupled single-phase flow in a fracture with heat conduction from a semi-infinite matrix (Figure 1). The solution to the same differential equations is given in Carslaw and Jaeger (1959). The thermal breakthrough in a geothermal reservoir can then be calculated from the solution for temperature as a function of time and distance along the fracture. The key assumptions are as follows:

1. Semi-infinite matrix in the x-direction
2. Heat conduction in the matrix is one-dimensional and perpendicular to the fracture
3. Thermal properties of rock are spatially and temporally constant
4. Single-phase liquid flow in a one-dimensional homogeneous fracture
5. Heat conduction in the fracture is neglected
6. Uniform initial temperature
7. Local thermodynamic equilibrium

When cold liquid water is injected into vapor-dominated geothermal reservoirs, the injected liquid phase will boil and transport in the vapor phase toward the producers. In additional to reducing the liquid velocity in the fractures, boiling also impacts the thermal velocity arising from cool liquid injection. Thus, the assumption of single-phase flow is not appropriate and leads to inaccurate predictions. Most naturally fractured reservoirs have finite fracture spacing and hence the assumption of semi-infinite matrix blocks is also not accurate.

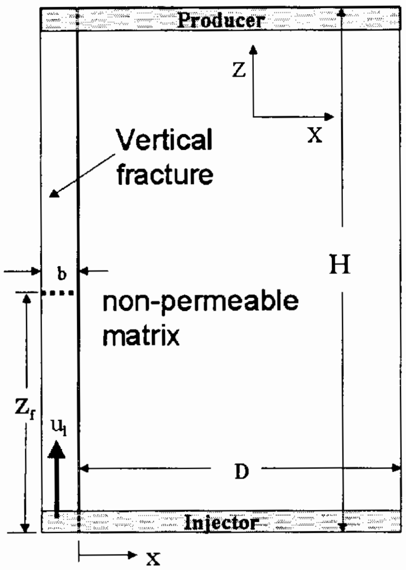

Figure 1. Schematic diagram of the single vertical fracture model.

Semi-Analytical Model with Finite Matrix

Single- Phase Liquid Flow

The same assumptions listed above have been made to derive a solution to the coupled single-phase flow in a fracture with heat conduction except this new solution is for a finite matrix rather than a semi-infinite matrix (the extension to two-phase flow is treated in the next section). The mass and energy balances for this problem are as follows:

The energy balance in the matrix is given by:

$$\frac{\partial T_m}{\partial t} = \frac{\lambda}{\rho_r C_r}\frac{\partial^2 T_m}{\partial x^2} \tag{1}$$

where Tm is the temperature in matrix, λ is the thermal conductivity of rock, ρ_r and C_r are density and thermal capacity of rock, respectively.

For single-phase flow in the fracture, the energy balance is:

$$M_{Tt}\frac{\partial T}{\partial t} + \rho_l \vec{u}_l C_{pl}\frac{\partial T}{\partial z} - \frac{\lambda}{b}\frac{\partial T_r}{\partial x}\bigg|_{x=0} = 0 \tag{2}$$

$$M_{Tt} = \phi\rho_l C_{pl} + (1-\phi)\rho_r C_{pr}$$

where M_{Tt} is the overall volumetric thermal capacity of fracture; ϕ is the porosity of the fracture, ρ_l and C_{pl} are density and thermal capacity of liquid phase water, respectively; and $\vec{u}_l$ is the superficial velocity of the injected liquid.

The initial and boundary conditions are:

$$T_f(x,0) = T_m(x,z,0) = T_o\ ;\ T(0,t) = T_J\ ;$$

$$T_f(z,t) = T_m(z,0,t)\ ;\ \frac{\partial T_m}{\partial x}\bigg|_{x=D} = 0$$

T_0 and T_J are the initial temperature and injection temperature, respectively. Subscripts f and m denote for the fracture and the matrix. T denotes temperature while t is for time.

Defining the following dimensionless variables:

$$T_D = \frac{T - T_o}{T_J - T_o}\ ;\quad X_D = \frac{x}{D}\ ;\quad Z_D = \frac{z}{b}\ ;$$

$$t_D = \frac{\lambda t}{\rho_r C_{pr} D^2} = \frac{\alpha_R t}{D^2}\ ;\quad \theta = \frac{M_{Tt} b}{\rho_r C_{pr} D}\ ;\quad \psi_l = \frac{D\rho_l C_{pl} u_l}{\lambda}$$

The following are the dimensionless form of the governing equations:

$$\frac{\partial T_{Dm}}{\partial t_D} = \frac{\partial^2 T_{Dm}}{\partial X^2} \tag{3}$$

$$\theta\frac{\partial T_D}{\partial t_D} + \Psi_1\frac{\partial T_D}{\partial Z_D} - \frac{\partial T_{Dm}}{\partial X_D}\bigg|_{X_D=0} = 0 \tag{4}$$

and the initial and boundary conditions are:

$$T_D(X_D,0) = T_{Dr}(X_D,Z_D,0) = 0$$

$$T_{fl}(0,t_D) = 1 \text{ for } t_D \geq 0\ ;\ T_D(Z_D,t_D) = T_{Dr}(Z_D,0,t_D)$$

$$\frac{\partial T_{Dr}}{\partial X_D}\bigg|_{X_D=1} = 0$$

The dimensionless form of the partial differential equation can be solved in Laplace space and is given below. The temperature in the fracture is:

$$u = \frac{f(z_D)}{s} \tag{5}$$

and in the matrix:

$$v = \frac{f(z_D)}{s}\left[\cosh\left(\sqrt{s}X_D\right) - \sinh\left(\sqrt{s}X_D\right)\tanh\left(\sqrt{s}\right)\right] \tag{6}$$

s is the Laplace variable, and

$$f(z_D) = \exp\left[-\frac{\sqrt{s}\tanh\left(\sqrt{s}\right) + \theta s}{\psi_l}Z_D\right] \tag{7}$$

Heat Transfer from the Matrix

The heat flux at the fracture-matrix interface is given by following equation:

$$q(t) = -\lambda\frac{\partial T_r}{\partial x}\bigg|_{x=0} \tag{8}$$

The cumulative heat flux at time t may be found by integrating above equation:

$$Q = \int_0^t q(t)dt \tag{8}$$

These can be written in the following dimensionless forms.

$$q_D = \frac{q(t)D}{\lambda(T_J - T_0)} \tag{9}$$

$$Q_D = \frac{Q(t)}{(T_0 - T_J)\rho_r C_{pr} D} \tag{10}$$

Equations 10 and 11 can be solved in Laplace space as given below.

$$q_D(s) = \frac{\sqrt{s}\tanh(\sqrt{s})}{s} f(z_D) \tag{11}$$

$$Q_D(s) = \frac{\sqrt{s}\tanh(\sqrt{s})}{s^2} f(z_D) \tag{12}$$

Energy Production and Thermal Recovery

In geothermal reservoirs with fractures, the energy is mainly stored in the matrix, and the fractures provide the flow conduit for the fluids. Selecting the injection temperature as the reference temperature, the total energy in the reservoir can be calculated as:

$$E_R = \rho_r C_{pr} LH\left[D + (1-\phi)b\right](T_0 - T_J) \tag{13}$$

The heat extraction rate or thermal power output by the produced fluid (liquid water) between T_w and T_J is:

$$e_p = m_p C_{pw}(T_w - T_J) \tag{14}$$

where m_p is the mass production rate of the liquid phase. In Laplace space, the cumulative energy production is:

$$E_p(s) = \frac{m_p C_{pw}(T_0 - T_J)}{s^2}\frac{D^2}{\alpha_R}\left[1 - f\left(\frac{H}{b}\right)\right] \tag{16}$$

where $\frac{H}{b}$ is the total dimensionless distance in the z direction from the injection point.

The thermal recovery of the geothermal reservoir is defined as the ratio of the amount of energy extracted by the injected mass, m_I, to the initial energy content in the reservoir. In Laplace space the thermal recovery can be calculated as follows:

$$\Re(s) = \frac{m_I C_{pw} D^2}{\lambda L^2\left[D + (1-\phi)b\right]}\frac{1}{s^2}\left[1 - f\left(\frac{H}{b}\right)\right] \tag{17}$$

In most cases, $D \gg (1-\phi)b$, so we can simplify above equation as follows.

$$\Re(s) = \frac{m_I C_{pw} D}{\lambda L^2}\frac{1}{s^2}\left[1 - f\left(\frac{H}{b}\right)\right] \tag{18}$$

Two-Phase Flow (Steam-Liquid)

Assuming the two-phase front in the fracture is sharp, we can calculate the front location from a mass balance with the flow in the fracture divided into two regions. The first region from the injector to the front is the single-phase liquid region, and the second region is from the front to the producer filled with steam. In the first liquid phase region, the dimensionless temperature in the fracture and the matrix are identical to Eqn. (5) and (6), respectively. In the vapor phase region, following a procedure similar to the one used in the derivation of the liquid flow equation, we get following dimensionless temperatures in Laplace space.

$$u_v = \frac{\varpi}{s} f(z_D) \tag{19}$$

in the fracture, and

$$v_v = \frac{\varpi}{s} f(z_D)\left[\cosh(\sqrt{s}X_D) - \sinh(\sqrt{s}X_D)\tanh(\sqrt{s})\right] \tag{20}$$

for the matrix. ϖ is typically a very small positive number. It is the dimensionless initial temperature of the vapor phase at $X_D = 0$. If it is very small, physically it implies that the temperature is constant in the vapor phase region. In other words, if $\varpi = 0$, it means that there is no temperature drop during the vapor phase flow. Given that the vapor phase initially in place is displaced ahead of the boiling front, a constant temperature in the vapor is considered a reasonable assumption.

Since it is difficult to find the inverse Laplace transform analytically, the Stehfest algorithm (Stehfest, 1979) has been implemented. Knowing the temperature and mass production rate, it is possible to calculate the enthalpy production rate as $m_p C_p T$.

Comparison of Models and Discussion of Results

Table 1, overleaf, summarizes the key input data for the example calculations in this paper. For the first example,

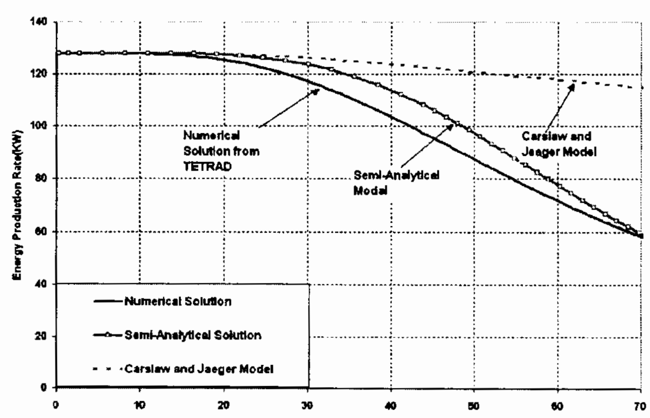

Figure 2. Energy production rates for the single vertical fracture model with single liquid phase: numerical (TETRAD) solution, infinite matrix (Carslaw and Jaeger), and the present study.

Table1. Input parameters for the numerical simulation and semi-analytical solution.

Property	Value	Property	Value
Thickness, [H] (m)	200	Rock density (kg/m^3)	2650
Length, [L] (m)	100	Rock heat capacity (kJ/kg°C)	1
Matrix Width, [D] (m)	20.48	Rock thermal conductivity (kJ/m°C.day)	249.2
Fracture Aperture, [b] (m)	0.05	Injection rate (kg/day)	11520
Initial Pressure (kPa)	5000	Producer bottom hole pressure (kPa)	3500
Initial Temperature (°C)	240	Injection temperature (°C)	35

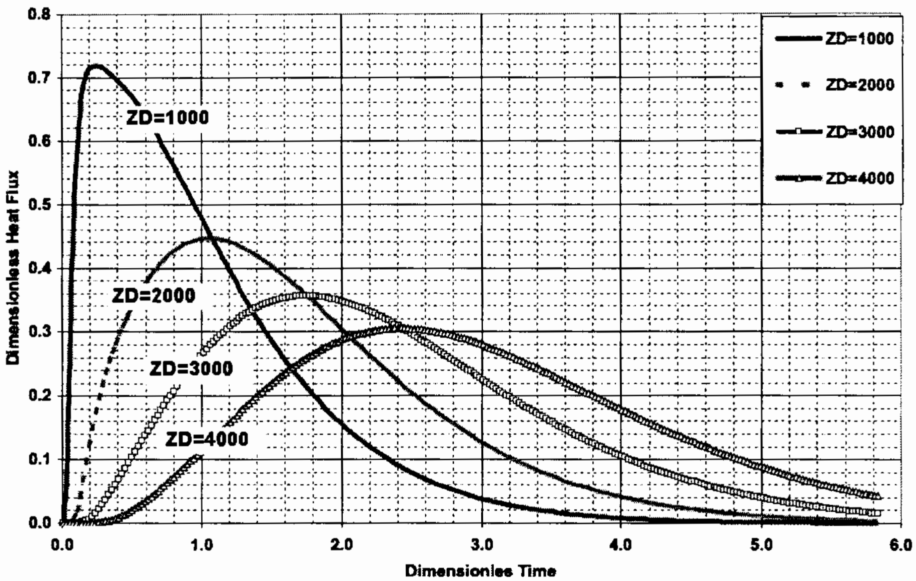

Figure 3. Dimensionless heat flux for the single vertical fracture model with single liquid phase at location of 50, 100, 150, and 200 m above the injection point.

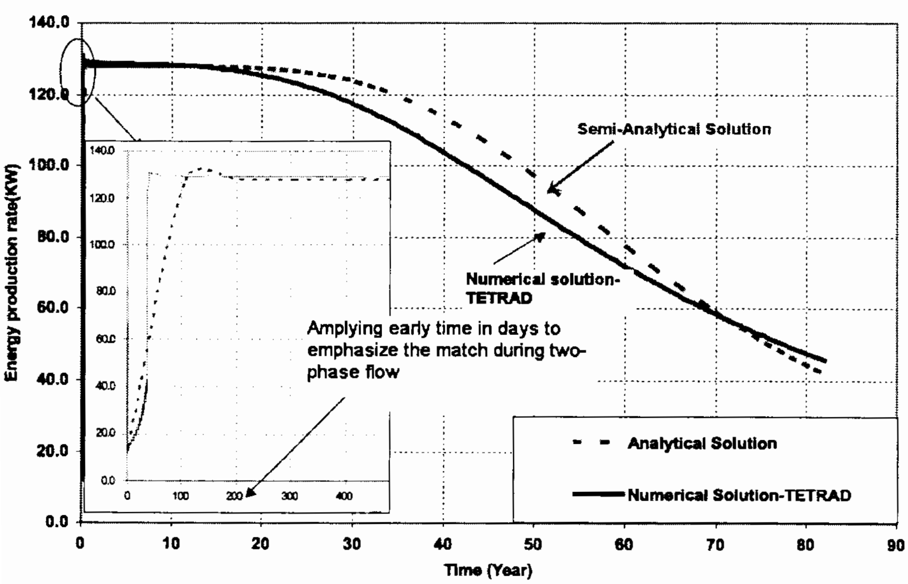

Figure 4. Comparison of numerical solution and semi-analytical solution for the two-phase flow in a single vertical model, the embedded picture is amplified part of early time in days.

both initial and producer bottom hole pressures are above the saturation pressure for 240°C so only liquid exists and flows in the fracture.

Figure 2 compares the energy production rate calculated using TETRAD with the semi-analytical solution as well as the analytical solution for a semi-infinite matrix. The latter model overestimates the energy production rate, while the numerical simulation and semi-analytical results are similar. Thus it can be concluded that for the purpose of energy depletion, the assumption of semi-infinite matrix block dimension is not a good approximation under these conditions.

Figure 3 shows the dimensionless heat flux as a function of dimensionless time at locations 50, 100, 150, and 200 meters above the injection point. The heat flux increases and then reaches a maximum before declining. Initially the heat flux is low because the at that point the temperature gradient is low even through the temperature is high; while the later declining period is caused by the low temperature difference between cold liquid and the cold fracture surface.

Figure 4 compares the energy production rate calculated using TETRAD with the semi-analytical solution for two-phase flow using the same parameters specified in Table 2 except the initial vapor saturation is 0.9999 (saturation conditions) and the producer bottom hole pressure is 2500 kPa. It shows that they match very well before the energy depletion starts. The embedded picture shows the energy comparison in early time to emphasize the early period of energy production. The calculated liquid breakthrough times and energy production predictions are similar.

Summary and Conclusions

A new solution to the coupled mass and energy flow in a fracture with heat conduction from the matrix has been derived in Laplace space. This new solution is more general than the solution in the literature because it is for a finite matrix rather than a semi-infinite matrix and because it accounts for two-phase flow in the fracture. The inversion of the Laplace transform is solved by numerical methods. Although not included in this paper, the semi-analytical method has been used to calculate type curves.

Predictions of energy production agree closely with those calculated from the numerical simulations with TETRAD

The pore volume of the fracture can be estimated from tracer data using the method of moments for a matrix with either zero or very low permeability.

Acknowledgements

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